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# Additional Experiments with the KHE20 Nurse Rostering Solver

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**Abstract** This paper is a supplement to 'KHE20: an improved solver for nurse rostering' [2]. It reports on additional experiments, which aim to show how helpful the various aspects of KHE20 are, and also whether some of KHE20's poorer results could be improved by allowing it to run for longer.

**Keywords** Nurse rostering · Time sweep · Ejection chains · XESTT

## 1 Introduction

This paper is a supplement to 'KHE20: an improved solver for nurse rostering' [2]. It reports on additional experiments, which aim to show how helpful the various aspects of KHE20 are, and also whether some of KHE20's poorer results could be improved by allowing it to run for longer.

## 2 Aspects of KHE20

This section investigates eleven aspects of KHE20, to see whether they make useful contributions. For each aspect, the results of running KHE20x8 with and without the aspect are compared. Only the Curtois original instances are tested, but they are very varied instances, so that if some aspect benefits the algorithm significantly, the results on them should make that clear.

If an omitted aspect consumes a significant amount of running time, the question arises as to whether its time should be reallocated to other aspects. There are arguments both ways. The tests are not consistent in this regard, and running time as well as cost must be considered when assessing the results.

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Each aspect tested is labelled in [2] by a two-character code. The algorithm variants are named using these codes. For example, aspect R2 is tested by comparing KHE20x8 with KHE20x8-R2, which is KHE20x8 with R2 omitted.

As explained in [2], the many small time limits cause otherwise identical runs to end at slightly different points, producing different results. However, the KHE20x8 columns in these tables are all the same. This is because, to save time, these experiments all start from an archive file containing the results of the experiment, reported on in the main paper, which runs KHE20x8 on the COI instances. The other columns contain fresh results.

The results appear in Tables 1-7. Each table has a two-paragraph caption. The first paragraph is the caption proper; the second evaluates the results.

At the foot of each table is a row of averages. The average costs can be misleading, when they are unduly affected by large constraint weights. So we have added a row showing the number of best solutions in each column. This provides an alternative basis for comparison which can be quite illuminating.

To summarize the results. Several of the aspects have turned out to be not useful. Testing all aspects on all archives might prove definitively that some of them could be removed. On the other hand, all aspects seem to be doing no harm, and, given the wide variation among instances, most of them probably do good sometimes. 'Biodiversity' produces robustness in algorithms as it does in the natural world, so the author is in no hurry to remove any aspects.

## 3 Running time

This section shows what KHE20 can do when its running time limits are doubled, from 2 seconds per day during construction, 2 minutes for the first repair phase, and 1 minute for the second repair phase, to 4 seconds, 4 minutes, and 2 minutes. Archive files `COI.xml`, `INRC2-8.xml`, and `CQ14.xml` are tested. The results appear in Tables 8-10.

To summarize the results. Doubling the running time does produce better solutions on some large instances. For example, the improvement in cost on instance COI-MER from 9282 to 8254 is almost half the distance to optimality. More often, however, the improvement is only slight.

In any case, the author does not advocate longer running times, because KHE20 is intended to run quickly. The real purpose of these experiments is to see what the current repertoire of ejection chain repair operations is capable of, when more time is available.

## References

1. Jeffrey H. Kingston, XESTT web site, `http://jeffreykingston.id.au/xestt` (2017)
2. Jeffrey H. Kingston, KHE20: an improved solver for nurse rostering. Submitted to PATAT 2020, also at `http://jeffreykingston.id.au/tt_papers/` (2020)
3. A. Legrain, J. Omer, and S. Rosat, A rotation-based branch-and-price approach for the nurse scheduling problem (2017). Working paper, available at `https://hal.archives-ouvertes.fr/hal-01545421`.

**Table 1** Evaluation of aspects of the KHE20 construction phase. The KHE20x8 column shows the results of KHE20x8 on the Curtois original instances; the KHE20x8-GR column shows the results of omitting task grouping; and the KHE20x8-CS column shows the results of omitting suppression of spurious costs during time sweep. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-aspects1.xml`, available at [1].

The differences are small. Omitting GR never produces a lower cost solution. Omitting CS produces a lower cost solution in six instances, but overall, KHE20x8 produces more best solutions. The effect of GR and CS was larger earlier in this project; as the ejection chain algorithm has improved, the need for them has diminished.

| **Instances (27)** | **KHE20x8** | | **KHE20x8-GR** | | **KHE20x8-CS** | |
|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 |
| COI-Musa | **175** | 1.1 | **175** | 0.8 | **175** | 0.8 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.0 |
| COI-LLR | 302 | 3.1 | 302 | 3.1 | **301** | 4.0 |
| COI-Azaiez | **0** | 0.8 | **0** | 0.8 | **0** | 0.9 |
| COI-GPost | 12 | 0.9 | 12 | 0.8 | **9** | 1.6 |
| COI-GPost-B | **5** | 1.0 | **5** | 1.0 | **5** | 0.7 |
| COI-QMC-1 | **16** | 2.7 | **16** | 2.8 | 18 | 2.5 |
| COI-QMC-2 | **29** | 5.3 | **29** | 5.3 | **29** | 5.1 |
| COI-WHPP | **3000** | 16.7 | **3000** | 13.4 | 3002 | 17.1 |
| COI-BCV-3.46.2 | **894** | 15.9 | **894** | 18.0 | **894** | 16.5 |
| COI-BCV-4.13.1 | **10** | 1.0 | **10** | 1.0 | **10** | 1.2 |
| COI-SINTEF | **0** | 0.5 | **0** | 0.7 | **0** | 0.4 |
| COI-ORTEC01 | 330 | 5.1 | 330 | 6.4 | **315** | 4.8 |
| COI-ORTEC02 | 300 | 7.2 | 300 | 7.5 | **280** | 10.5 |
| COI-ERMGH | **779** | 598.1 | 817 | 597.1 | 865 | 612.6 |
| COI-CHILD | **149** | 494.8 | 151 | 499.0 | 151 | 539.8 |
| COI-ERRVH | **2144** | 598.1 | 2250 | 598.1 | 2283 | 605.8 |
| COI-HED01 | 151 | 7.6 | 151 | 8.5 | **138** | 10.6 |
| COI-Valouxis-1 | **80** | 1.2 | **80** | 1.1 | 180 | 1.5 |
| COI-Ikegami-2.1 | **0** | 13.5 | **0** | 11.5 | **0** | 18.9 |
| COI-Ikegami-3.1 | **8** | 21.8 | **8** | 22.0 | 13 | 30.8 |
| COI-Ikegami-3.1.1 | **8** | 26.2 | **8** | 25.4 | 14 | 36.1 |
| COI-Ikegami-3.1.2 | **7** | 26.1 | **7** | 25.8 | 16 | 34.9 |
| COI-BCDT-Sep | **230** | 5.8 | **230** | 5.2 | 250 | 6.4 |
| COI-MER | 9347 | 577.6 | 9455 | 569.1 | **8971** | 567.6 |
| **Average** | 666 | 90.1 | 675 | 89.8 | **664** | 93.7 |
| **No. of best costs** | **21** | | 18 | | 17 | |

**Table 2** Evaluation of aspects of the KHE20 construction phase. The KHE20x8-CE column show the results of omitting ejection chain repair of limit resources defects after assigning each day; the KHE20x8-CR column shows the results of omitting resource rematching after assigning each day; and the KHE20x8-CI column shows the results of omitting resource rematching of each day individually at the end. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-aspects2.xml`, available at [1].

The averages are skewed by a lucky result for KHE20x8-CR on instance COI-WHPP (the difference is just one defect, but with cost 1000); adjusting for that, the results are similar.

| **Instances (27)** | **KHE20x8** | | **KHE20x8-CE** | | **KHE20x8-CR** | | **KHE20x8-CI** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.0 | **0** | 0.1 | **0** | 0.0 | **0** | 0.0 |
| COI-Musa | **175** | 0.8 | **175** | 1.4 | **175** | 0.9 | **175** | 0.8 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.0 |
| COI-LLR | 302 | 4.0 | 302 | 4.0 | **301** | 3.8 | 302 | 3.0 |
| COI-Azaiez | **0** | 0.8 | **0** | 0.8 | **0** | 0.7 | **0** | 0.7 |
| COI-GPost | 12 | 0.9 | 12 | 0.9 | **8** | 0.7 | 12 | 0.8 |
| COI-GPost-B | **5** | 1.0 | **5** | 1.0 | 8 | 0.7 | **5** | 1.0 |
| COI-QMC-1 | **16** | 2.7 | **16** | 2.6 | 18 | 2.0 | **16** | 3.0 |
| COI-QMC-2 | **29** | 4.3 | **29** | 3.9 | **29** | 3.9 | **29** | 4.2 |
| COI-WHPP | 3000 | 12.7 | 3000 | 12.5 | **2004** | 18.0 | 3000 | 12.5 |
| COI-BCV-3.46.2 | **894** | 16.0 | **894** | 15.7 | **894** | 16.5 | **894** | 15.4 |
| COI-BCV-4.13.1 | **10** | 1.3 | **10** | 1.0 | **10** | 0.9 | **10** | 1.0 |
| COI-SINTEF | **0** | 0.5 | **0** | 0.7 | **0** | 0.2 | **0** | 0.7 |
| COI-ORTEC01 | 330 | 5.3 | 330 | 5.4 | **305** | 5.2 | 330 | 5.3 |
| COI-ORTEC02 | **300** | 7.4 | **300** | 7.5 | 310 | 9.4 | **300** | 7.4 |
| COI-ERMGH | 808 | 598.0 | **779** | 597.1 | **779** | 594.0 | **779** | 514.3 |
| COI-CHILD | **149** | 494.1 | **149** | 499.6 | 151 | 474.0 | 151 | 491.3 |
| COI-ERRVH | **2148** | 597.2 | 2154 | 595.4 | 2155 | 587.4 | 2243 | 520.0 |
| COI-HED01 | **151** | 10.6 | **151** | 9.2 | 153 | 9.8 | **151** | 11.4 |
| COI-Valouxis-1 | 80 | 1.2 | 80 | 1.1 | **60** | 1.4 | 80 | 1.2 |
| COI-Ikegami-2.1 | **0** | 11.7 | **0** | 11.5 | **0** | 9.0 | **0** | 11.6 |
| COI-Ikegami-3.1 | **8** | 21.2 | **8** | 20.9 | 12 | 21.2 | **8** | 21.3 |
| COI-Ikegami-3.1.1 | **8** | 31.9 | **8** | 24.7 | 14 | 23.1 | 10 | 28.0 |
| COI-Ikegami-3.1.2 | **7** | 25.6 | **7** | 25.2 | 13 | 20.2 | 9 | 29.6 |
| COI-BCDT-Sep | **230** | 5.1 | **230** | 5.4 | 250 | 4.6 | **230** | 5.2 |
| COI-MER | 9282 | 566.4 | 9521 | 568.5 | 9357 | 559.2 | **8930** | 530.3 |
| **Average** | 665 | 89.7 | 673 | 89.5 | **630** | 87.7 | 654 | 82.2 |
| **No. of best costs** | **20** | | **20** | | 16 | | 18 | |

**Table 3** Evaluation of edge adjustment during time sweep. The KHE20x8-CW column shows the results of omitting to favour resources that have more available workload; the KHE20x8-CL column shows the results of omitting to favour assignments that bring fewer constraints from below their maximum limits to their maximum limits; and KHE20x8-CB shows the results of omitting to favour shorter runs of consecutive assignments. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-aspects3.xml`, available at [1].

Although the differences are not large, KHE20x8 does have the lowest average cost and the largest number of best solutions.

| **Instances (27)** | **KHE20x8** | | **KHE20x8-CW** | | **KHE20x8-CL** | | **KHE20x8-CB** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.0 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Musa | **175** | 0.8 | **175** | 1.1 | **175** | 1.4 | **175** | 1.5 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-LLR | 302 | 4.0 | 302 | 5.3 | **301** | 4.1 | 302 | 3.3 |
| COI-Azaiez | **0** | 0.8 | **0** | 1.3 | **0** | 0.8 | **0** | 1.0 |
| COI-GPost | 12 | 0.9 | **8** | 1.0 | 10 | 1.0 | 12 | 0.8 |
| COI-GPost-B | **5** | 1.0 | 7 | 1.2 | 9 | 1.5 | **5** | 1.1 |
| COI-QMC-1 | 16 | 2.7 | **15** | 2.4 | 17 | 1.8 | 16 | 2.1 |
| COI-QMC-2 | **29** | 4.3 | **29** | 4.6 | **29** | 5.4 | **29** | 4.1 |
| COI-WHPP | **3000** | 12.7 | 3002 | 17.1 | 3001 | 15.9 | **3000** | 12.6 |
| COI-BCV-3.46.2 | **894** | 16.0 | **894** | 23.2 | **894** | 16.1 | **894** | 21.0 |
| COI-BCV-4.13.1 | **10** | 1.3 | **10** | 1.2 | **10** | 1.2 | **10** | 1.1 |
| COI-SINTEF | **0** | 0.5 | **0** | 0.7 | **0** | 0.6 | **0** | 0.5 |
| COI-ORTEC01 | 330 | 5.3 | **305** | 4.5 | 325 | 4.6 | 330 | 5.1 |
| COI-ORTEC02 | 300 | 7.4 | **295** | 6.3 | 300 | 9.4 | 300 | 7.1 |
| COI-ERMGH | 808 | 598.0 | **779** | 601.1 | **779** | 600.7 | **779** | 598.0 |
| COI-CHILD | **149** | 494.1 | **149** | 489.0 | 150 | 489.6 | 151 | 494.6 |
| COI-ERRVH | **2148** | 597.2 | 2172 | 594.6 | 2258 | 594.3 | 2255 | 598.0 |
| COI-HED01 | 151 | 10.6 | **147** | 9.9 | 155 | 8.0 | 151 | 11.4 |
| COI-Valouxis-1 | **80** | 1.2 | 160 | 1.8 | 160 | 0.9 | **80** | 1.1 |
| COI-Ikegami-2.1 | **0** | 11.7 | **0** | 18.4 | **0** | 12.5 | **0** | 11.6 |
| COI-Ikegami-3.1 | 8 | 21.2 | 8 | 23.5 | **7** | 23.9 | 8 | 21.2 |
| COI-Ikegami-3.1.1 | **8** | 31.9 | 11 | 22.9 | 11 | 25.2 | **8** | 25.4 |
| COI-Ikegami-3.1.2 | **7** | 25.6 | 10 | 22.9 | 9 | 31.3 | **7** | 25.9 |
| COI-BCDT-Sep | **230** | 5.1 | 240 | 5.6 | **230** | 4.1 | **230** | 5.2 |
| COI-MER | **9282** | 566.4 | 9653 | 563.5 | 9298 | 568.7 | 9408 | 567.4 |
| **Average** | **665** | 89.7 | 680 | 89.8 | 671 | 89.8 | 672 | 89.7 |
| **No. of best costs** | **19** | | 17 | | 14 | | 17 | |

**Table 4** Evaluation of aspects of the KHE20 repair phases. The KHE20x8-R2 column shows the results of omitting the second repair phase; KHE20x8-RM shows the results of omitting resource rematching from both repair phases; and KHE20x8-EE shows the results of omitting ejection chain repair from both repair phases. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-aspects4.xml`, available at [1].

This shows that ejection chains have a major impact, and that the second repair phase is also important on some instances (notably COI-BCDT-Sep, where the amount of time spent on repair is not an issue). A good result for XHE20x8-RM on one instance (COI-MER) is responsible for its low average cost. It may indicate that, for some instances, time spent on resource rematching might be better spent on ejection chains.

| **Instances (27)** | **KHE20x8** | | **KHE20x8-R2** | | **KHE20x8-RM** | | **KHE20x8-EE** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.0 | **0** | 0.0 | **0** | 0.0 | **0** | 0.1 |
| COI-Musa | **175** | 0.8 | **175** | 0.8 | **175** | 0.8 | **175** | 0.1 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 | 300 | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.1 |
| COI-LLR | 302 | 4.0 | 302 | 3.0 | **301** | 4.1 | 343 | 0.2 |
| COI-Azaiez | **0** | 0.8 | **0** | 0.7 | **0** | 1.0 | 14135 | 1.1 |
| COI-GPost | **12** | 0.9 | **12** | 0.5 | **12** | 0.8 | 23 | 0.2 |
| COI-GPost-B | **5** | 1.0 | **5** | 0.8 | 7 | 1.0 | 56 | 0.2 |
| COI-QMC-1 | **16** | 2.7 | **16** | 2.1 | 17 | 2.7 | 3039 | 0.7 |
| COI-QMC-2 | **29** | 4.3 | **29** | 2.5 | **29** | 2.7 | **29** | 2.8 |
| COI-WHPP | **3000** | 12.7 | 3002 | 13.3 | 3001 | 12.8 | 5007 | 0.6 |
| COI-BCV-3.46.2 | **894** | 16.0 | **894** | 6.7 | **894** | 17.1 | 898 | 2.8 |
| COI-BCV-4.13.1 | **10** | 1.3 | **10** | 0.6 | **10** | 0.9 | 11 | 0.6 |
| COI-SINTEF | **0** | 0.5 | **0** | 0.6 | **0** | 0.4 | 6 | 0.4 |
| COI-ORTEC01 | 330 | 5.3 | 330 | 3.4 | **320** | 4.4 | 426 | 0.7 |
| COI-ORTEC02 | **300** | 7.4 | **300** | 5.9 | **300** | 5.4 | 450 | 0.9 |
| COI-ERMGH | 808 | 598.0 | **779** | 479.2 | **779** | 597.7 | 803 | 417.9 |
| COI-CHILD | **149** | 494.1 | 153 | 384.2 | 151 | 497.8 | 152 | 307.2 |
| COI-ERRVH | **2148** | 597.2 | 2365 | 476.8 | 2251 | 599.7 | 2352 | 417.2 |
| COI-HED01 | **151** | 10.6 | 4287 | 7.6 | **151** | 7.3 | 163 | 3.5 |
| COI-Valouxis-1 | **80** | 1.2 | **80** | 0.9 | **80** | 1.0 | 2120 | 0.9 |
| COI-Ikegami-2.1 | **0** | 11.7 | **0** | 9.7 | 1 | 3.5 | **0** | 11.4 |
| COI-Ikegami-3.1 | **8** | 21.2 | **8** | 12.1 | **8** | 6.8 | **8** | 20.6 |
| COI-Ikegami-3.1.1 | **8** | 31.9 | 14 | 14.7 | 12 | 10.6 | **8** | 24.7 |
| COI-Ikegami-3.1.2 | **7** | 25.6 | **7** | 18.6 | 14 | 8.8 | **7** | 24.4 |
| COI-BCDT-Sep | **230** | 5.1 | 650 | 1.5 | 250 | 5.8 | 380 | 1.7 |
| COI-MER | 9282 | 566.4 | 10163 | 451.4 | **8764** | 573.5 | 12164 | 392.7 |
| **Average** | 665 | 89.7 | 873 | 70.3 | **649** | 87.7 | 1595 | 60.5 |
| **No. of best costs** | **23** | | 18 | | 18 | | 8 | |

**Table 5** Evaluation of aspects of KHE20's ejection chain repair. The KHE20x8-EW column shows the results of omitting widening; KHE20x8-ER shows the results of omitting reversing; and KHE20x8-EB shows the results of omitting balancing. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-aspects5.xml`, available at [1].

KHE20x8 is clearly best here, producing the lowest average cost and the largest number of best solutions. Omitting balancing speeds up the algorithm significantly and is beneficial in a few instances, but it is clearly inferior overall.

| **Instances (27)** | **KHE20x8** | | **KHE20x8-EW** | | **KHE20x8-ER** | | **KHE20x8-EB** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.0 | **0** | 0.0 | **0** | 0.0 | **0** | 0.1 |
| COI-Musa | **175** | 0.8 | **175** | 0.3 | **175** | 0.8 | **175** | 0.1 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | 300 | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.1 |
| COI-LLR | 302 | 4.0 | 302 | 3.0 | **301** | 1.7 | 302 | 2.1 |
| COI-Azaiez | **0** | 0.8 | **0** | 0.7 | **0** | 1.0 | 5 | 1.0 |
| COI-GPost | 12 | 0.9 | 11 | 0.4 | **9** | 0.7 | 18 | 0.5 |
| COI-GPost-B | **5** | 1.0 | 8 | 0.6 | 8 | 0.7 | 13 | 0.6 |
| COI-QMC-1 | **16** | 2.7 | 18 | 0.9 | 23 | 1.6 | 18 | 1.3 |
| COI-QMC-2 | **29** | 4.3 | **29** | 2.0 | **29** | 3.8 | **29** | 1.3 |
| COI-WHPP | **3000** | 12.7 | 3001 | 5.7 | 3001 | 16.0 | 3001 | 4.3 |
| COI-BCV-3.46.2 | **894** | 16.0 | 896 | 7.5 | **894** | 14.1 | 895 | 5.4 |
| COI-BCV-4.13.1 | **10** | 1.3 | **10** | 0.7 | **10** | 0.6 | **10** | 0.5 |
| COI-SINTEF | **0** | 0.5 | **0** | 0.4 | **0** | 0.4 | 2 | 0.4 |
| COI-ORTEC01 | **330** | 5.3 | 350 | 2.1 | 345 | 4.6 | 370 | 1.2 |
| COI-ORTEC02 | **300** | 7.4 | 330 | 3.9 | 345 | 8.4 | 360 | 2.5 |
| COI-ERMGH | **808** | 598.0 | 958 | 582.5 | 830 | 589.0 | 946 | 538.0 |
| COI-CHILD | **149** | 494.1 | 153 | 452.2 | **149** | 498.9 | 860 | 155.5 |
| COI-ERRVH | **2148** | 597.2 | 2479 | 570.0 | 2253 | 594.9 | 3203 | 337.5 |
| COI-HED01 | 151 | 10.6 | **147** | 5.7 | 153 | 5.8 | **147** | 4.3 |
| COI-Valouxis-1 | 80 | 1.2 | **60** | 0.6 | 80 | 1.2 | **60** | 1.1 |
| COI-Ikegami-2.1 | **0** | 11.7 | **0** | 7.3 | **0** | 12.5 | **0** | 4.2 |
| COI-Ikegami-3.1 | 8 | 21.2 | **7** | 13.8 | 13 | 26.9 | **7** | 8.2 |
| COI-Ikegami-3.1.1 | **8** | 31.9 | 12 | 15.7 | 18 | 33.8 | 13 | 7.5 |
| COI-Ikegami-3.1.2 | 7 | 25.6 | **6** | 15.3 | 17 | 32.6 | 8 | 10.8 |
| COI-BCDT-Sep | 230 | 5.1 | **220** | 2.8 | 270 | 3.4 | 260 | 2.8 |
| COI-MER | 9282 | 566.4 | 8855 | 556.5 | 9387 | 567.4 | **8655** | 569.1 |
| **Average** | **665** | 89.7 | 668 | 83.4 | 678 | 89.7 | 728 | 61.5 |
| **No. of best costs** | **19** | | 14 | | 13 | | 10 | |

**Table 6** Evaluation of widening during the KHE20 ejection chain repair algorithm. The KHE20x8-EW2 column shows the results of widening up to 2 tasks; KHE20x8-EW4 shows the results of widening up to 4 tasks (this is what KHE20x8 does); and so on. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-apsects6.xml`, available at [1].

A lucky result on instance COI-WHPP has skewed the average cost for KHE20x8-EW2. Adjusting for that, KHE20x8 seems to be doing about the right amount of widening.

| **Instances (27)** | **KHE20x8-EW2** | | **KHE20x8-EW4** | | **KHE20x8-EW6** | | **KHE20x8-EW8** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.0 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Musa | **175** | 0.5 | **175** | 1.3 | **175** | 1.5 | **175** | 1.2 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-LLR | **302** | 3.1 | **302** | 4.4 | **302** | 4.1 | **302** | 4.3 |
| COI-Azaiez | **0** | 0.9 | **0** | 1.1 | **0** | 0.8 | **0** | 0.9 |
| COI-GPost | **9** | 0.8 | 12 | 0.8 | 12 | 0.8 | 12 | 0.9 |
| COI-GPost-B | 6 | 0.9 | **5** | 1.0 | **5** | 1.3 | **5** | 1.3 |
| COI-QMC-1 | **16** | 1.3 | **16** | 3.4 | **16** | 4.1 | **16** | 6.5 |
| COI-QMC-2 | **29** | 3.0 | **29** | 4.0 | **29** | 4.6 | **29** | 4.3 |
| COI-WHPP | **2002** | 11.8 | 3000 | 14.8 | 3000 | 13.2 | 3000 | 13.4 |
| COI-BCV-3.46.2 | **894** | 14.0 | **894** | 20.8 | **894** | 18.3 | **894** | 19.3 |
| COI-BCV-4.13.1 | **10** | 0.8 | **10** | 1.0 | **10** | 1.0 | **10** | 1.2 |
| COI-SINTEF | **0** | 0.4 | **0** | 0.8 | **0** | 0.7 | **0** | 0.5 |
| COI-ORTEC01 | 335 | 4.9 | 330 | 6.2 | **310** | 5.4 | **310** | 5.1 |
| COI-ORTEC02 | 310 | 5.4 | **300** | 9.2 | **300** | 7.4 | **300** | 7.2 |
| COI-ERMGH | **779** | 596.5 | **779** | 598.5 | **779** | 599.4 | **779** | 599.2 |
| COI-CHILD | 150 | 482.1 | **149** | 490.3 | 150 | 500.4 | **149** | 499.7 |
| COI-ERRVH | 2248 | 591.4 | **2140** | 595.3 | 2261 | 596.4 | 2155 | 599.0 |
| COI-HED01 | **147** | 6.2 | 151 | 7.7 | 151 | 9.8 | 151 | 10.6 |
| COI-Valouxis-1 | **60** | 0.9 | 80 | 1.2 | 80 | 1.3 | 80 | 1.2 |
| COI-Ikegami-2.1 | **0** | 9.9 | **0** | 15.0 | **0** | 11.3 | **0** | 14.9 |
| COI-Ikegami-3.1 | **7** | 18.3 | 8 | 20.5 | 8 | 22.6 | 8 | 21.8 |
| COI-Ikegami-3.1.1 | 9 | 17.7 | **8** | 25.1 | **8** | 25.5 | **8** | 23.2 |
| COI-Ikegami-3.1.2 | **7** | 21.3 | **7** | 25.1 | **7** | 26.1 | **7** | 26.1 |
| COI-BCDT-Sep | **230** | 4.3 | **230** | 5.4 | **230** | 5.2 | **230** | 5.2 |
| COI-MER | 9875 | 565.7 | **9431** | 570.5 | 9863 | 565.4 | 9874 | 575.4 |
| **Average** | **652** | 87.5 | 669 | 89.8 | 689 | 89.9 | 685 | 90.5 |
| **No. of best costs** | 20 | | **21** | | 19 | | 20 | |

**Table 7** Evaluation of balancing during the KHE20 ejection chain repair algorithm. The KHE20x8-EB3 column shows the results of trying 3 balancing repairs; KHE20x8-EB6 tries 6 balancing repairs: and so on (KHE20x8 tries 12). This table is derived from XESTT archive file `KHE20-2020-01-20-COI-apsects7.xml`, available at [1].

There is some evidence here that KHE20x8 is doing too much balancing, although most of the difference in average cost between KHE20x8-EB3 and KHE20x8-EB12 is due to one instance (COI-MER). This area needs more study. Balancing is likely to be of more benefit to some instances—those whose workload limits are tight—than others. For solves that reach the time limit, balancing repairs lack variety compared with other repairs that could be tried in the limited time.

| **Instances (27)** | **KHE20x8-EB3** | | **KHE20x8-EB6** | | **KHE20x8-EB12** | | **KHE20x8-EB24** | |
|---|---|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | 0.1 | **0** | 0.0 | **0** | 0.1 | **0** | 0.1 |
| COI-Musa | **175** | 0.8 | **175** | 0.8 | **175** | 1.4 | **175** | 0.8 |
| COI-Millar-2.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | 0.1 | **0** | 0.0 | **0** | 0.1 | **0** | 0.1 |
| COI-LLR | **302** | 3.9 | **302** | 3.9 | **302** | 4.9 | **302** | 3.3 |
| COI-Azaiez | **0** | 1.0 | **0** | 0.8 | **0** | 1.0 | **0** | 1.4 |
| COI-GPost | **9** | 0.6 | **9** | 0.8 | 12 | 1.0 | 12 | 0.8 |
| COI-GPost-B | 9 | 0.7 | **5** | 1.0 | **5** | 1.2 | **5** | 1.0 |
| COI-QMC-1 | **16** | 1.7 | 17 | 2.3 | **16** | 4.3 | **16** | 2.7 |
| COI-QMC-2 | **29** | 3.3 | **29** | 4.7 | **29** | 6.8 | **29** | 3.8 |
| COI-WHPP | **3000** | 14.4 | 3001 | 13.7 | **3000** | 15.6 | **3000** | 12.8 |
| COI-BCV-3.46.2 | **894** | 12.6 | **894** | 18.6 | **894** | 21.3 | **894** | 19.0 |
| COI-BCV-4.13.1 | **10** | 0.8 | **10** | 1.0 | **10** | 1.7 | **10** | 1.0 |
| COI-SINTEF | **0** | 0.6 | **0** | 0.5 | **0** | 0.8 | **0** | 0.5 |
| COI-ORTEC01 | 330 | 4.1 | **320** | 3.9 | 330 | 7.5 | 330 | 5.0 |
| COI-ORTEC02 | **290** | 7.2 | 310 | 6.6 | 300 | 10.2 | 300 | 7.2 |
| COI-ERMGH | 819 | 595.5 | **779** | 597.8 | **779** | 599.4 | 799 | 599.7 |
| COI-CHILD | 151 | 426.6 | **149** | 484.8 | 152 | 498.2 | **149** | 495.5 |
| COI-ERRVH | **2127** | 569.5 | 2149 | 601.2 | 2145 | 595.6 | 2239 | 594.8 |
| COI-HED01 | **143** | 8.9 | 155 | 13.7 | 151 | 7.8 | 151 | 7.9 |
| COI-Valouxis-1 | **40** | 0.9 | 80 | 1.5 | 80 | 1.3 | 80 | 1.4 |
| COI-Ikegami-2.1 | **0** | 12.8 | **0** | 18.1 | **0** | 13.3 | **0** | 13.7 |
| COI-Ikegami-3.1 | 10 | 19.8 | **6** | 28.6 | 8 | 20.9 | 8 | 20.9 |
| COI-Ikegami-3.1.1 | 10 | 24.8 | 11 | 38.1 | **8** | 25.6 | **8** | 27.8 |
| COI-Ikegami-3.1.2 | 10 | 25.1 | 10 | 38.9 | **7** | 25.8 | **7** | 24.7 |
| COI-BCDT-Sep | 260 | 3.4 | **230** | 5.6 | **230** | 5.5 | **230** | 5.3 |
| COI-MER | **8438** | 567.4 | 9836 | 572.7 | 9505 | 571.5 | 9657 | 575.5 |
| **Average** | **632** | 85.4 | 684 | 91.1 | 672 | 90.5 | 682 | 89.9 |
| **No. of best costs** | **19** | | 18 | | 18 | | 18 | |

**Table 8** Solving the Curtois original instances with double running time. The Misc column shows the solutions from Curtois' web site; the KHE20x8 column shows KHE20x8's results; and the KHE20x8-T2 column shows KHE20x8's results when its running time limits are doubled. This table is derived from XESTT archive file `KHE20-2020-01-20-COI-time.xml`, available at [1].

Only four of the COI instances are affected by the changed limits. Except for COI-CHILD, where KHE20x8's solution is already optimal, all four show a clear reduction in cost.

| **Instances (27)** | **Misc** | | **KHE20x8** | | **KHE20x8-T2** | |
|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| COI-Ozkarahan | **0** | - | **0** | 0.0 | **0** | 0.1 |
| COI-Musa | **175** | - | **175** | 0.8 | **175** | 0.8 |
| COI-Millar-2.1 | **0** | 1.0 | **0** | 0.1 | **0** | 0.1 |
| COI-Millar-2.1.1 | **0** | - | **0** | 0.1 | **0** | 0.1 |
| COI-LLR | **301** | 10.0 | 302 | 4.0 | 302 | 3.7 |
| COI-Azaiez | **0** | 600.0 | **0** | 0.8 | **0** | 1.1 |
| COI-GPost | **5** | - | 12 | 0.9 | 12 | 0.8 |
| COI-GPost-B | **3** | - | 5 | 1.0 | 5 | 1.0 |
| COI-QMC-1 | **13** | - | 16 | 2.7 | 16 | 2.9 |
| COI-QMC-2 | **29** | - | **29** | 4.3 | **29** | 3.9 |
| COI-WHPP | **5** | - | 3000 | 12.7 | 3000 | 12.6 |
| COI-BCV-3.46.2 | **894** | 17840.0 | **894** | 16.0 | **894** | 16.1 |
| COI-BCV-4.13.1 | **10** | - | **10** | 1.3 | **10** | 1.0 |
| COI-SINTEF | **0** | - | **0** | 0.5 | **0** | 0.4 |
| COI-ORTEC01 | **270** | 105.0 | 330 | 5.3 | 330 | 5.2 |
| COI-ORTEC02 | **270** | - | 300 | 7.4 | 300 | 7.5 |
| COI-ERMGH | **779** | 124.0 | 808 | 598.0 | **779** | 1189.5 |
| COI-CHILD | **149** | - | **149** | 494.1 | **149** | 952.5 |
| COI-ERRVH | **2001** | - | 2148 | 597.2 | 2123 | 1159.5 |
| COI-HED01 | **136** | - | 151 | 10.6 | 151 | 7.8 |
| COI-Valouxis-1 | **20** | - | 80 | 1.2 | 80 | 1.2 |
| COI-Ikegami-2.1 | **0** | 13.0 | **0** | 11.7 | **0** | 12.4 |
| COI-Ikegami-3.1 | **2** | 21600.0 | 8 | 21.2 | 8 | 21.7 |
| COI-Ikegami-3.1.1 | **3** | 2820.0 | 8 | 31.9 | 8 | 25.8 |
| COI-Ikegami-3.1.2 | **3** | 2820.0 | 7 | 25.6 | 7 | 26.0 |
| COI-BCDT-Sep | **100** | - | 230 | 5.1 | 230 | 5.4 |
| COI-MER | **7081** | 36002.7 | 9282 | 566.4 | 8254 | 1135.4 |
| **Average** | **454** | | 665 | 89.7 | 625 | 170.2 |

**Table 9** Solving the Second International Timetabling Competition 8-week instances with double running time. The LOR17 column shows the results from [3]; the KHE20x8 column shows KHE20x8's results; and the KHE20x8-T2 column shows KHE20x8's results when its running time limits are doubled. This table is derived from XESTT archive file `KHE20-2020-01-20-INRC2-8-time.xml`, available at [1].

There is some improvement in cost, but the results are still uncompetitive with those of [3]. It seems that running time is not the issue here. This area needs more study.

| **Instances (20)** | **LOR17** | | **KHE20x8** | | **KHE20x8-T2** | |
|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| INRC2-8-030-1-27093606 | **2125** | - | 2695 | 140.4 | 2695 | 161.4 |
| INRC2-8-030-1-67535629 | **1735** | - | 2255 | 121.1 | 2255 | 127.7 |
| INRC2-8-035-0-62987798 | **2570** | - | 3745 | 178.3 | 3630 | 239.4 |
| INRC2-8-035-1-08161720 | **2330** | - | 3505 | 180.7 | 3390 | 269.4 |
| INRC2-8-040-0-06892664 | **2635** | - | 3700 | 197.6 | 3635 | 306.9 |
| INRC2-8-040-2-50487172 | **2495** | - | 3605 | 197.1 | 3595 | 214.0 |
| INRC2-8-050-1-17857418 | **4990** | - | 6110 | 338.6 | 6030 | 451.0 |
| INRC2-8-050-1-97538831 | **5000** | - | 5865 | 307.3 | 5850 | 438.6 |
| INRC2-8-060-0-62990813 | **2425** | - | 3670 | 328.6 | 3520 | 452.5 |
| INRC2-8-060-2-10340391 | **2590** | - | 3920 | 364.3 | 3785 | 491.8 |
| INRC2-8-070-0-33923752 | **4660** | - | 6010 | 394.2 | 5820 | 672.7 |
| INRC2-8-070-0-93072110 | **4770** | - | 6175 | 393.7 | 6065 | 649.8 |
| INRC2-8-080-1-44993605 | **4225** | - | 6020 | 547.4 | 5840 | 785.4 |
| INRC2-8-080-2-04091962 | **4495** | - | 6400 | 546.5 | 6155 | 909.6 |
| INRC2-8-100-0-01789154 | **2145** | - | 3900 | 490.3 | 3710 | 732.7 |
| INRC2-8-100-1-24793928 | **2250** | - | 4240 | 517.8 | 4020 | 778.5 |
| INRC2-8-110-0-21172647 | **4010** | - | 5290 | 495.7 | 5140 | 834.1 |
| INRC2-8-110-0-32494137 | **3575** | - | 4805 | 470.0 | 4645 | 742.7 |
| INRC2-8-120-0-09945103 | **2670** | - | 4455 | 476.3 | 4350 | 836.6 |
| INRC2-8-120-1-72645202 | **3125** | - | 4890 | 515.1 | 4755 | 924.1 |
| **Average** | **3241** | | 4563 | 360.0 | 4444 | 550.9 |

**Table 10** Solving the Curtois and Qu (2014) instances with double running time. The CQ-GR column shows results obtained by Curtois using Gurobi; the KHE20x8 column shows KHE20x8's results; and the KHE20x8-T2 column shows KHE20x8's results when its running time limits are doubled. This table is derived from XESTT archive file `KHE20-2020-01-20-CQ14-time.xml`, available at [1].

Only seven of the instances for which KHE20x8 finds feasible solutions have been able to utilize the extra time (CQ14-13 to CQ14-19), and only two of them (CQ14-13 and CQ14-17) have benefited significantly from it.

| **Instances (24)** | **CQ-GR** | | **KHE20x8** | | **KHE20x8-T2** | |
|---|---|---|---|---|---|---|
| | **Cost** | **Time** | **Cost** | **Time** | **Cost** | **Time** |
| CQ14-01 | **607** | - | **607** | 0.3 | **607** | 0.3 |
| CQ14-02 | **828** | - | **828** | 2.5 | **828** | 2.5 |
| CQ14-03 | **1001** | - | **1001** | 2.8 | **1001** | 2.9 |
| CQ14-04 | **1716** | - | 1720 | 3.2 | 1720 | 3.1 |
| CQ14-05 | **1143** | - | 1239 | 10.1 | 1239 | 8.2 |
| CQ14-06 | **1950** | - | 2066 | 13.9 | 2066 | 12.7 |
| CQ14-07 | **1056** | - | 1078 | 24.0 | 1078 | 23.7 |
| CQ14-08 | **1323** | - | 1429 | 62.1 | 1429 | 72.1 |
| CQ14-09 | **439** | - | 453 | 91.1 | 453 | 91.8 |
| CQ14-10 | **4631** | - | 4665 | 103.2 | 4665 | 102.8 |
| CQ14-11 | **3443** | - | 3457 | 130.7 | 3457 | 131.4 |
| CQ14-12 | **4040** | - | 4081 | 147.1 | 4081 | 149.2 |
| CQ14-13 | **1388** | - | 2205 | 385.4 | 1844 | 739.7 |
| CQ14-14 | **1280** | - | 1414 | 294.8 | 1414 | 324.4 |
| CQ14-15 | **4039** | - | 4649 | 360.9 | 4632 | 591.6 |
| CQ14-16 | **3233** | - | 3766 | 316.3 | 3763 | 348.7 |
| CQ14-17 | **5851** | - | 6410 | 360.9 | 6207 | 721.1 |
| CQ14-18 | **4760** | - | 5503 | 360.7 | 5484 | 717.5 |
| CQ14-19 | **3218** | - | 3461 | 361.8 | 3436 | 721.7 |
| **Average** | **2418** | | 2633 | 159.6 | 2600 | 250.8 |
| CQ14-20 | | | inf. | 369.9 | **inf.** | 727.3 |
| CQ14-21 | | | inf. | 386.7 | **inf.** | 747.1 |
| CQ14-22 | | | inf. | 378.2 | **inf.** | 734.5 |
| CQ14-23 | **17428** | - | inf. | 420.5 | inf. | 780.6 |
| CQ14-24 | **48777** | - | inf. | 578.5 | inf. | 929.3 |